HOMEWORK SET 7: 3-D SCHRÖDINGER EQUATION I
Due Monday, February 10, 2025

PROBLEMS FROM TZDII[1]

1) 8.43 The probability of finding an electron in the region r > a is $\int_a^\infty P(r)dr$. What is the probability that a 1s electron in hydrogen would be found outside the Bohr Radius, a_B? (LOOK UP R_{1s} IN TABLE 8.2, FOLLOW INSTRUCTIONS IN THE FOOTNOTE OF THE TABLE TO WRITE THE PROBABILITY DENSITY, P(R), THEN INTEGRATE BY PARTS. DOES YOUR ANSWER MAKES SENSE? MAKE A PHYSICAL ARGUMENT AS TO WHY IT DOES OR DOES NOT.)

2) 8.47 a) Write down the Θ equation (8.65) for the 2p states with m = ±1. Show that the solution is Θ(θ) = sin(θ). This means that the complete wave functions for the 2p states with m = ±1 are

$$\psi_{2,1,\pm1} = R_{2p}(r)\sin(\theta)e^{\pm i\phi}$$

b) Prove that the sum of these two wave functions is the $2p_x$ wave function (times an uninteresting factor of 2) and that the difference is the $2p_y$ function (times 2i). [Hint: Rewrite $e^{\pm i\phi}$ as $\cos(\phi) \pm i\sin(\phi)$ and remember the relations for x and y in terms of r, θ, and ϕ in Fig. 8.11.]. Comment on what this means in a radial potential.

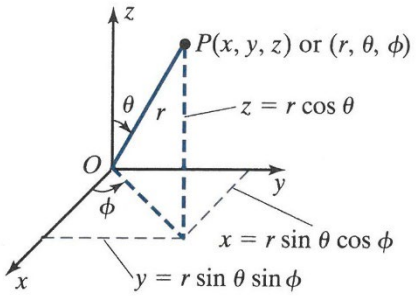
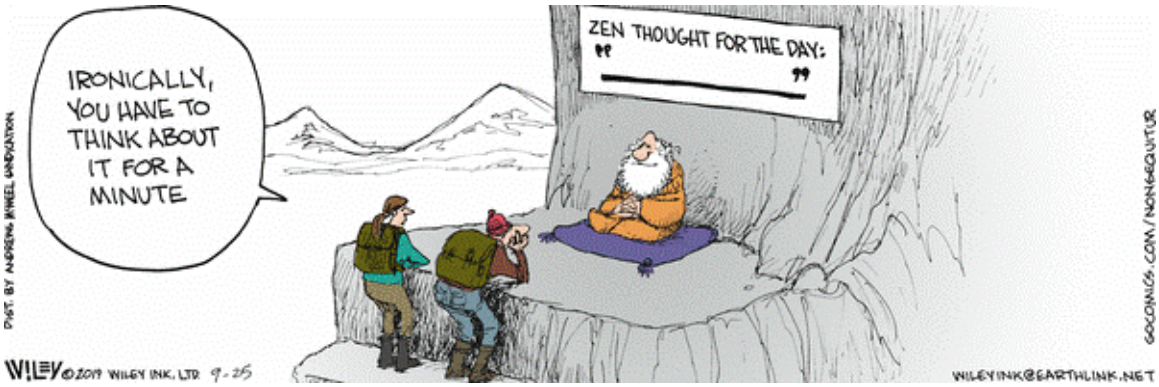


FIGURE 8.11

The spherical polar coordinates of a point P are (r, θ, ϕ), where r is the distance OP, θ is the angle between OP and the z axis, and ϕ is the angle between the x-z plane and the vertical plane containing OP.



[1] Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Editon, Pearson, Prentice Hall, 2004